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Hedging Performance and Basis Risk in Stock Index Futures

STEPHEN FIGLEWSKI*

IN EARLY 1982, TRADING BEGAN at three different exchanges in futures contracts based on stock indexes. Stock index futures were an immediate success, and quickly led to a proliferation of new futures and options markets tied to various indexes. One reason for this success was that index futures greatly extended the range of investment and risk management strategies available to investors by offering them, for the first time, the possibility of unbundling the market and nonmarket components of risk and return in their portfolios. Many portfolio management and other hedging applications in investment banking and security trading have been described elsewhere¹ ranging from use by a passive fund manager to reduce risk over a long time horizon to use by an underwriter to hedge the market risk exposure in a stock offering for one or two days.

In considering the potential applications of index futures, it is clear that in nearly every case a cross-hedge is involved. That is, the stock position that is being hedged is different from the underlying portfolio for the index contract.² This means that return and risk for an index futures hedge will depend upon the behavior of the "basis," i.e., the difference between the futures price and the cash price. Hedging a position in stock will necessarily expose it to some measure of basis risk—risk that the change in the futures price over time will not track exactly the value of the cash position.

Basis risk can arise from a number of different sources, and is a more significant problem for stock index contracts than for other financial futures, like Treasury bills and bonds.³ The most apparent cause of basis risk is the nonmarket component of return on the cash stock position. Since the index contract is tied to the behavior of an underlying stock market index, nonmarket risk cannot be hedged. This is the essential problem of a cross-hedge. However, basis risk can be present even when the hedge involves a position in the index portfolio itself and there is no nonmarket risk. For one thing, returns to the index portfolio include dividends, while the index, and the index future, only track the capital value of the portfolio. Any risk associated with dividends on the portfolio will become basis risk in a hedged position. Still, dividends are fairly low and also quite stable, so this may not be a terribly important shortcoming.


Much more important than dividend risk is the fact that the futures price is not directly tied to the underlying index, except for the final settlement price on

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¹ See, for example, Figlewski and Kon [1982].

² In fact, in the case of the Value Line futures contract traded on the Kansas City Board of Trade, there is no stock portfolio which exactly duplicates the index.

³ See Elton, Gruber, and Rentzler [1983] or Hoag and Labarge [1983].



the expiration date. Day to day fluctuations in the difference between them induce fluctuations in the returns on a hedged position. The magnitude of this risk, which is in addition to nonmarket and dividend risk, is limited by the possibility of arbitrage between cash and futures markets. In markets where transactions costs are small and arbitrage is straightforward, as in Treasury bills, basis risk may be negligible. But for stock index futures, a perfect arbitrage appears to be infeasible. The Standard and Poor's 500 index, for instance, contains 500 stocks in precise proportions. It is impossible to assemble such a portfolio in a reasonable size and to buy or sell all of the stocks simultaneously in order to capitalize on short run deviations of the index futures price from its theoretical level, especially if it is necessary to sell them all short. Instead, arbitrage as it is done in this market is essentially risk-arbitrage. A trading portfolio consisting of a small subset (perhaps fifty or so) of the stocks in the index is selected and traded against the futures contract when discrepancies become too large.⁴ Because the trade is not risk free and there are sizable transactions costs, the range within which the futures price can move fairly freely without inducing arbitrage trading is broad enough to allow substantial basis risk.

This paper examines the basis and the different sources of basis risk on the Standard and Poor's 500 index contract. We develop a number of results about the use and usefulness of stock index futures in hedging and about the behavior of this new market as it has evolved. The next section presents a simplified theory of hedging in the presence of basis risk and displays the risk-return combinations that could have been achieved in practice by hedging several broadly diversified stock portfolios with S&P 500 futures. Section 3 discusses the sources of basis risk in a hedge involving the Standard and Poor's portfolio itself. We consider the effects of dividend risk, the length of the holding period, and the time remaining to expiration of the futures contract. In Section 4, we examine the movement in the basis over time, which determines the return to a hedged portfolio. We begin with a discussion of the equilibrium futures price, based on arbitrage with the cash market, and then examine empirically how well the theory describes the level and dynamics of actual futures prices. We find a clear distinction between the behavior of the market in its early months and currently. The final section summarizes the results and suggests some implications about contract design for stock index futures.

II. Hedging with Stock Index Futures

Individual stocks and all stock portfolios, except for those specifically designed to have zero beta, are exposed to some market risk. In this section we will first discuss in theoretical terms how a single futures contract based on a broad market index can be used to hedge market risk due to price fluctuation. We will then examine the returns and risk on actual hedged portfolios.⁵

⁴ Other strategies, like placing newly available funds selectively into stocks or a combination of stock index futures and Treasury bills according to the relative pricing of the two are also employed, especially by institutional investors.

⁵ We restrict the analysis to strategies involving a constant hedge ratio. The evidence presented below suggests that hedge performance may be improved further by use of a dynamic strategy.

Let us begin by defining the random variable returns on the portfolio to be hedged, \tilde{R}_p , the spot index, \tilde{R}_I , and the index futures contract, \tilde{R}_F , assuming a holding period of length T .

$$\tilde{R}_p = \frac{\tilde{V}_T - V_0 + \tilde{D}_p}{V_0} \tag{1}$$

where V_0 and \tilde{V}_T denote the beginning and ending market values for the portfolio. \tilde{D}_p represents the cumulative value as of T of the dividends paid out on the portfolio during the period, assuming reinvestment at the riskless rate of interest from the date of payout until T . The dividend payout is a random variable because the amount, its timing, and the reinvestment rate are all uncertain as of time 0.

The return on the index portfolio is

$$\tilde{R}_I = \frac{\tilde{I}_T - I_0 + \tilde{D}_I}{I_0} \tag{2}$$

where variables are defined analogously to (1).

The rate of return on a futures contract is not a well-defined concept, since taking a futures position does not require an initial outlay of capital.⁶ For expository convenience we will define the rate of return on futures as the change in the futures price divided by the initial level of the spot index:

$$\tilde{R}_F = \frac{\tilde{F}_T - F_0}{I_0} \tag{3}$$

Expressing this in terms of the basis, i.e., the futures price minus the spot index:

$$\begin{aligned} \tilde{R}_F &= \frac{\tilde{I}_T - I_0 + \tilde{D}_I}{I_0} - \frac{\tilde{D}_I}{I_0} + \frac{(\tilde{F}_T - \tilde{I}_T) - (F_0 - I_0)}{I_0} \\ \tilde{R}_F &= \tilde{R}_I - \frac{\tilde{D}_I}{I_0} + \frac{\tilde{B}_T - B_0}{I_0} \end{aligned} \tag{4}$$

The rate of return on a stock index futures contract is equal to the total return on the underlying index portfolio, minus the dividend yield on the index, plus the change in the basis over the period as a fraction of the initial index.

Now consider the return on a hedged portfolio in which futures contracts on N index “shares” have been sold short against the long portfolio of stocks. An index share is defined to be an amount of the index portfolio whose market value is equal to \$1 times the spot index. Most currently traded stock index futures have contract sizes of 500 index shares.

$$\begin{aligned} \tilde{R}_H &= \frac{(\tilde{V}_T - V_0 + \tilde{D}_p) - N(\tilde{F}_T - F_0)}{V_0} = \tilde{R}_p - \left(\frac{NI_0}{V_0}\right) \times \left(\frac{\tilde{F}_T - F_0}{I_0}\right) \\ \tilde{R}_H &= \tilde{R}_p - h\tilde{R}_F, \end{aligned} \tag{5}$$

⁶ The initial margin deposit to open a futures position does not represent an investment of capital since it can be posted in the form of interest bearing Treasury bills.

where h , the hedge ratio, is the current value of the index shares sold forward as a fraction of the current value of the portfolio being hedged. h determines the overall risk and return characteristics of the hedged position, which are given by the customary portfolio formulas:

$$\bar{R}_h = \bar{R}_p - h\bar{R}_f \quad (6)$$

$$\sigma_h^2 = \sigma_p^2 + h^2\sigma_f^2 - 2h\sigma_{pF}. \quad (7)$$

Bars represent expectations, σ with a single subscript denotes a variance and σ with two subscripts a covariance.

To find the constant hedge ratio which minimizes risk, we set the derivative of (7) with respect to h equal to zero and obtain

$$h^* = \sigma_{pF}/\sigma_F^2 \quad (8)$$

This is easily computed in practice by simply running a regression of R_p on R_F using historical data. The slope coefficient in the equation is h^* . One might think of it as the beta of the portfolio with respect to the futures contract.

Substituting into (7) yields the variance of returns for the minimum risk hedge,

$$\sigma_{\text{MIN}}^2 = \sigma_p^2 (1 - \rho_{pF}^2) \quad (9)$$

where ρ_{pF} is the correlation coefficient between the returns on the stock portfolio and the futures contract. It is apparent that only with perfect correlation can risk be completely eliminated by hedging.

Looking back to the definition of \bar{R}_F in eq. (4), we see that the variance of futures returns is influenced by three random variables: total returns on the market index portfolio, dividends on the market, and the change in the basis between the future and its underlying index. These will naturally all affect the risk minimizing hedge ratio as well.

In the special case where dividends are not random and the hedge is to be held until the futures contracts expire, so that the change in the basis is also nonstochastic, the effect of these terms disappears, leaving

$$h^* = \sigma_{pI}/\sigma_I^2 = \beta_p. \quad (10)$$

The risk minimizing hedge ratio in this special case is the portfolio's beta coefficient with respect to the market index. Earlier discussion of hedging with index futures suggested using the portfolio's beta as the appropriate hedge ratio. While dividends tend to be relatively stable, the same is not true of the basis, which is quite volatile over short periods. Hence using beta as the hedge ratio is unlikely to be optimal, except when the position is to be held until maturity of the futures.

To examine how effective stock index futures hedges would have been in reality, we have calculated the risk and return combinations which could have been achieved by selling Standard and Poor's 500 futures against the underlying portfolios of five major stock indexes over one week holding periods. The sample period was from June 1, 1982 through September 30, 1983. The indexes were the Standard and Poor's 500 index itself, the New York Stock Exchange composite, the American Stock Exchange composite, the National Association of Securities

Dealers Automated Quotation System (NASDAQ) index of over-the-counter stocks, and the Dow Jones Industrials index. These are all diversified portfolios—meaning nonmarket risk is substantially smaller than for individual stocks—but they are different in character from one another.

The first two are market value weighted portfolios containing, respectively, 500 and about 1500 of the largest capitalization stocks. Either is a good proxy for the “market” portfolio of financial theory and should contain very little nonmarket risk. The AMEX and OTC indexes are also value weighted and well diversified, within their segments of the market, but both contain stocks of smaller companies which move somewhat independently of the S&P index. Finally, the Dow Jones portfolio contains only 30 stocks of very large firms, weighted by their market prices. But despite its different composition, the Dow portfolio is significantly more closely correlated with the S&P than are the AMEX or OTC.

The first, and most difficult, step was to construct series of weekly returns, including dividends, on the five portfolios. Dividends are a problem because the payout pattern is very lumpy within a quarter. This might impart potentially important, and unhedgeable, variation in returns over short holding periods. Dividends were treated differently for each portfolio.

For the S&P portfolio itself we constructed a dividend inclusive series from data on the actual dividends paid on all 500 stocks over the sample period. This series is analyzed below when we look at the importance of systematic dividend risk. Dividends for the NYSE portfolio were assumed to follow the same time pattern as on the S&P, and were scaled to give the same dividend yield. The AMEX index includes dividends, treating them as if they were reinvested in the portfolio as they accrued, so no dividend adjustment was necessary. Without data on the payout pattern for the OTC index, we were forced to assume dividends were paid continuously over the sample period at an annual rate equal to 87 percent of the rate on the NYSE index, the historical value for the relative payouts on these two indexes.⁷ Finally, for the Dow we used a series which took the actual payout on the index portfolio and smoothed it evenly over the quarter. Thus, dividend risk has been eliminated from the OTC and Dow portfolios, but dividend yield remains.

The futures contract was in all cases the Standard and Poor’s 500 future nearest to expiration, assuming a rollover to the next contract at expiration. We confined the analysis to the near contract because preliminary research showed that there was not very much difference between the hedging properties of the nearest and the second contract. Also nearly all trading volume is in the near month so that liquidity is much greater in that contract. Returns on futures were computed as in equation (3).

Finally, since in theory the return to a fully hedged portfolio (or more generally, one containing only unsystematic risk) should be equal to the riskless interest rate, for comparison we constructed a series of holding period yields on a “riskless” asset. For this purpose we chose three month bank CD’s, assuming they were

⁷ See Ibbotsen and Fall [1979].

held for one week and sold in the secondary market at a price that yielded the market interest rate at that time.

Table 1 shows the results of one week hedges taking both the risk minimizing h^* from eq. (8) and the portfolio's beta as hedge ratios. The first two columns show annualized mean returns and standard deviations on the unhedged portfolios. The next three give the hedge ratio, mean return and standard deviation for the minimum risk hedged position, while the final columns provide the same information for positions with the portfolio beta as hedge ratio. Note that both h^* and beta have been calculated from the sample data. In an actual hedging application, these would have to be estimated, subject to sampling error, from past data.

Consider first the hedge involving the S&P portfolio itself. Clearly there is no nonmarket risk here to reduce hedging effectiveness. Risk reduction is significant, from a standard deviation of 19.0% on the unhedged portfolio down to 4.6% for the minimum risk hedge. Return also drops substantially, to 9.7%. Comparing these results to the riskless asset, we see that while hedging could have reduced portfolio risk by about 76 percent, even when hedging the underlying index portfolio, residual returns variability was still well above that for money market securities, while mean returns were lower.

Reasonably good risk reduction was also achieved for the NYSE and the Dow portfolios and returns were also slightly higher. Hedging of the portfolios of smaller stocks, however, was distinctly less good. In both cases, residual risk amounted to about 60 percent of the unhedged standard deviation. Returns on the hedged positions were well above the riskless rate, but less than half of what they would have been without hedging.

When the portfolio betas were used as the hedge ratios, hedge performance deteriorated. In all cases, beta hedges were dominated by the minimum risk hedges, which had both lower risk and higher return. Table 1 shows that the problem is not so much the higher risk on beta hedges—the standard deviations are not very different—but the substantially lower returns. Of course, had stock prices been down over this period, positions with a larger hedge ratio would have

Table 1
Effectiveness of Standard and Poor's 500 Index Futures in Hedging Major Stock Index Portfolios Over one Week Holding Period

Portfolio	Unhedged		Minimum Risk Hedge			Beta Hedge		
	\bar{R}	σ	h^*	\bar{R}_h^*	σ_h^*	β	\bar{R}_β	σ_β
S&P 500	39.1	19.0	.85	9.7	4.6	1.00	4.7	5.6
NYSE	39.3	18.3	.82	11.3	5.0	.96	6.3	5.9
AMEX	45.3	20.9	.78	18.5	12.4	.94	13.0	12.8
OTC	46.5	17.6	.64	24.4	10.9	.77	20.0	11.2
DOW	39.4	19.4	.86	9.9	5.8	.99	5.3	6.4
Riskless								
Asset	10.8	0.9	—	—	—	—	—	—
S&P 500 Futures	34.4	21.6	—	—	—	—	—	—

Notes: Mean returns and standard deviations are in percent, at annual rates.

Sample period: 6/1/82–9/30/83.

earned higher returns. But since the stock market goes up on average over the long run, this result will be the exception rather than the rule.

The results in Table 1 can be summarized as follows. For a one week holding period, hedging a diversified portfolio weighted toward large capitalization stocks can yield fairly good risk reduction, to about 20 to 30 percent of the unhedged portfolio's standard deviation. Returns are in the neighborhood of those on riskless assets. However, hedging effectiveness is substantially reduced by the presence of unsystematic risk, even in the amount contained in a broadly diversified portfolio of small stocks like the AMEX and OTC portfolios. (The R^2 for regressions of their returns on the S&P were .73 and .69, respectively, well above those for individual stocks.) This suggests that a short duration hedge for an individual stock or a small portfolio might be quite unsatisfactory. Finally, because of basis risk, the minimum risk hedge ratio was less than the portfolio's beta in every case, with the adverse effects of overhedging being more serious for returns than for risk levels.

III. The Components of Basis Risk

Table 1 showed that hedging performance for stock index futures is significantly affected by basis risk. Nonmarket risk on the stock portfolio being hedged is especially important, even in the amount present in a well diversified portfolio of small capitalization stocks. Moreover, basis risk was not negligible even when the cash portfolio being hedged was the underlying index portfolio itself. What is the source of basis risk on the Standard and Poor's portfolio?

One reason which we have already mentioned for the futures and cash price movements to deviate from one another is dividends. Unanticipated variation in dividends can induce variability in hedged portfolio returns because the futures price tracks the capital value of the index portfolio, adjusting for the expected dividend payout up to maturity.

In general, basis risk arises simply because the connection between the futures market and the cash market is imperfect, except at maturity of the futures contract. How closely tied they will be at any one time depends on how far prices are allowed to deviate from their equilibrium relationship before arbitrageurs begin to take positions in the two markets to earn excess returns. Because of the difficulty of doing a true arbitrage in stock index futures, fairly wide discrepancies can arise and persist in these markets. But they can not become arbitrarily large, while returns variability increases directly with the length of the time interval considered. This means that basis risk as a fraction of total risk should decrease as the holding period is extended, and hedging effectiveness should improve. A one week hedge ought to be better than an overnight hedge and not as good as a one month hedge.

The attractiveness of an arbitrage trade will also depend on the length of time the position must be held before the profit will be realized. Since the basis must go to zero at expiration, but is uncertain before then, it stands to reason that the closer the future is to maturity, the smaller its deviations from the equilibrium value will be. This suggests that hedging effectiveness may go up as the future gets close to expiration.

Table 2
Effects of Various Determinants of Basis Risk on Hedging Performance

Determinant of Risk	Portfolio	Unhedged		Minimum Risk Hedge		
		\bar{R}	σ	h^*	\bar{R}_h^*	σ_h^*
Dividends	S&P 500	39.1	19.0	.85	9.75	4.62
	S&P 500 without dividends	32.8	19.1	.86	3.31	4.48
Holding Period	1 day	37.4	17.3	.67	14.18	8.13
	20 days	39.4	13.5	.83	10.80	5.49
Time to Futures Expiration	0-1 month	41.5	17.6	.85	13.80	3.66
	1-2 months	24.9	20.7	.87	12.27	3.91
	2-3 months	52.1	19.3	.84	3.16	5.96

Notes: See notes to Table 1.

Except where specified, holding periods are all 5 trading days.

Table 2 provides some information on these determinants of basis risk in hedges involving the S&P 500 portfolio. For comparison, the first line duplicates the results for one week hedges that were reported in Table 1. In the second line, we examine the effects of dividend variability by considering hedging just the capital value of the index portfolio. The mean capital gain return on the unhedged portfolio is 32.8 and its standard deviation of return is 19.1 percent, which is only a tiny bit different from the risk on the overall portfolio. The results for risk minimizing hedges show that dividend risk is of little importance. The hedge ratio is the same as in Line 1, hedged returns differ only by the amount of the dividend yield, and residual risk is reduced by a very small amount.

This leaves us with "noise" in the price relationship between the cash and futures markets as the primary source of basis risk. Noise may come from many sources. For example, transitory imbalances in supply and demand associated with large orders in one market or the other can lead to temporary price distortions. So can expectations differences between futures and cash market traders. Some noise is also due to the fact that reported prices the two markets are not perfectly synchronous.⁸ Clearly, noise from nonsynchronous prices does not represent risk in the same sense that noise from other sources does, but there is no way to separate it out in our study.

Noise from all of these sources as a fraction of total returns variability should decrease with the length of hedge duration. The next two lines in Table 2 show hedge performance for overnight hedges and positions held for 20 trading days, i.e. four weeks. As we expected, overnight hedges are not as effective as one week hedges in reducing risk. Because of the relatively larger amount of basis risk in 1 day returns, the risk minimizing hedge ratio is lower than in Line 1 and the amount of risk left in the fully hedged portfolio is well above that for the longer duration. However, when we turn to four week hedges, we do not find any

⁸ There are two reasons for this. First, the futures exchange closes fifteen minutes later than the stock exchange, so that closing prices are not simultaneous quotes. More important, the "closing" prices for many of the smaller stocks included in the index are last trade prices, which may actually have occurred some time before the close of the market. This means that the closing S&P index is not the true level of the index at the close, i.e., the level based on prices at which the component stocks could have been traded.

improvement in performance. The standard deviation of hedge returns is considerably higher than for one week hedges, especially when compared with the low estimate for the standard deviation on the unhedged portfolio at this horizon. Possibly sampling error is playing a role here, since there are only 17 observations for four week hedges. We can think of no way to explain this result as an equilibrium relationship.

In the last three lines we examine the effect of time to expiration on hedging effectiveness. The hedge returns for one week holding periods were split into three sets by time to maturity. Taking subperiods led to samples with different returns experience (and also fewer data points in each). Risk on the unhedged portfolios was quite similar across subperiods, however. In comparing hedge performance, we see little difference between 0 to 1 month to expiration and 1 to 2 months. Minimum risk hedge ratios are very close and so are hedge returns, despite the substantial difference in returns on the index portfolio in these subsamples. (Eliminating such differences, of course, is the objective of hedging.) It does appear that the nearer to maturity hedges have somewhat lower risk, but this is not actually a valid comparison. Due to the difference in risk on the unhedged portfolios, the percent of risk reduction for the two was actually 79 and 81 percent, respectively.

Hedging effectiveness seems to be reduced when one goes beyond two months to expiration. The hedge ratio is lower, indicating that the relation between price movements in the two markets is less close, and unhedged risk is higher. Risk reduction is about 69 percent at this horizon. Further, mean returns are considerably lower than for hedges taken closer to expiration. This is a result of differences in the pricing of the futures contracts relative to their equilibrium values according to time to expiration. We will examine this phenomenon more closely in the next section.

IV. Explaining the Basis

While fluctuation in the basis causes risk in a hedged portfolio, the mean change in the basis over the holding period determines mean returns. This section describes the pricing relationship for index futures which sets the equilibrium value for the basis, and shows how well the theoretical model fits actual prices. We then present some evidence on the dynamics of the basis, which shows, among other things, that the market's behavior has changed over time as it has matured.

Consider the investment strategy of buying the S&P index portfolio, selling an equal amount of S&P futures against it, and holding the position until the futures contract expires. The hedge fixes the capital gain on the portfolio at $F_0 - I_0$, and there will also be a flow of dividends over time. Since the position is effectively riskless (assuming dividend risk to be negligible), it should earn the riskless rate of interest. This yields the following theoretical equilibrium value for the futures price to prevent profitable arbitrage

$$F_t^e = I_t e^{r(T-t)} - \sum_{\tau=t+1}^T D(\tau) e^{r(T-\tau)} \quad (11)$$

Table 3
Differences Between Theoretical and Actual Futures Prices

Sample Period	Position	Price Difference			Arbitrage Return		
		Mean	Standard Deviation	t-Stat on H_0	Mean	Standard Deviation	t-stat on H_0
Full Sample	1st	.21	1.00	4.00	.03	11.25	0.05
	2nd	-.08	1.43	1.03	-.47	3.33	2.66
First Third	1st	-.13	1.30	1.08	-2.88	13.07	2.27
	2nd	-1.16	1.64	7.53	-2.87	4.07	7.52
Middle Third	1st	.38	.90	4.55	2.12	9.19	2.46
	2nd	.52	1.10	4.98	.89	2.21	4.28
Last Third	1st	.39	.60	6.87	2.54	6.23	4.20
	2nd	.42	.71	6.23	.69	1.21	6.02

NOTES: 1st position has 0-3 months to expiration, 2nd position 3-6 months.

Arbitrage return is the annualized excess return over the riskless rate earned by a position that is long the S&P index portfolio and short an equal amount of S&P futures.

H_0 is the hypothesis that the true mean price difference or arbitrage return is zero.

where T is the maturity date and the second term is the cumulative value of dividends paid, assuming reinvestment at the riskless interest rate up to date T .

Equation (11) takes no account of marking to market of the futures, essentially treating it as a forward contract. Although the theoretical importance of marking to market has been discussed,⁹ efforts to determine the economic significance of this factor suggest that it is rather small.¹⁰ Another factor left out of this equation is the value of the tax timing option discussed by Cornell and French [1983]. Although there is no empirical evidence available on the magnitude of this, we suspect that it is probably not large.

Table 3 provides data on how closely actual futures prices conformed to their theoretical values during our sample period. Two measures of the price discrepancy are reported. The first is the actual price difference, $F_t^e - F_t$. This does not take into account the time period over which this deviation must disappear nor the amount of money which would have to be tied up in an arbitrage portfolio in order to earn a riskless excess return from the mispricing. Thus we also report the excess return at an annual rate which would be earned by the strategy of buying the index portfolio, selling futures against it, and holding to maturity. This strategy earns an excess return which is positive if the future is overpriced and negative if it is underpriced. Results are given for both the nearest (1st position) and next nearest (2nd position) contracts to expiration, including the t -statistic on the hypothesis that there is no significant difference between actual and theoretical prices. The sample is then split into thirds to show the differences in the market's behavior over time. One of the early puzzles of the stock index futures market was that futures prices were typically well below their theoretical levels, often even falling below the spot index itself.¹¹ Our results show that underpricing of futures was significant in the first third of the sample, but this was not true of the period as a whole for the nearest contracts (which have by

⁹ See Richard and Sundaresan [1981] or Cox, Ingersoll, and Ross [1981], for example.

¹⁰ See Elton, Gruber, and Rentzler [1983] or Hill, Schneeweis, and Mayerson [1982].

¹¹ See Figlewski [1983] for a discussion of the various explanations of this phenomenon offered at the time, as well as a prediction that it would prove to be transitory.

far the largest trading interest). The actual price difference was in fact significantly positive, while the excess arbitrage return was essentially zero. The second position contract was underpriced on average, though significantly so only for arbitrage returns.

Table 3 shows that average deviations from equilibrium futures prices were in general statistically significant, and also that even in the most recent period, excess returns to arbitrage trading (if it could be done economically) were substantial. The fact that the standard deviation on arbitrage returns for the nearest contract was 6.23 percent in the last third of the sample suggests that there were many attractive opportunities. It is important to recognize that the standard deviation does not measure risk in this case. Every nonzero arbitrage return was a profit which could have been earned risklessly. The standard deviation simply measures how these amounts varied from day to day. The fact that prices today do not stray as far from their equilibrium levels as they once did, suggests that the market has become more efficient as it has developed.

If a deviation from the equilibrium futures prices is simply due to noise, arbitrage trading should tend to push it back into line. But if mispricing is the result of specific factors like the tax timing option, it should persist over long periods. Whether, and how quickly the basis moves toward its theoretical level can yield some insight into the causes of futures mispricing.

Early experience suggested that mispricing in the same direction persisted for long periods, which pointed toward a structural explanation. Another apparent feature of futures pricing was that the basis tended to widen (futures rose relative to the spot index) when the market was rising, and it shrank, even to negative values, when the market dropped. The alleged reason for this behavior was that the futures market was overreacting to the forces which moved the cash market. Although this explanation is rather unsatisfying, it is hard to find one that is consistent with market efficiency.

To examine the dynamics of the basis, we ran a regression of the change in the basis on the deviation from the theoretical value and on the change in the spot index,

$$B_t - B_{t-1} = C_0 + C_1(B_t^e - B_{t-1}) + C_2(I_t - I_{t-1}) \quad (12)$$

Table 4 shows the results of this regression over the whole sample and during the three subperiods used in Table 3. For the whole sample, the estimated coefficients on both explanatory variables were highly significant. On average the basis tended to move 43 percent of the way from yesterday's actual level to today's equilibrium value, and the basis did tend to increase when the market rose and decrease when it fell. Explanatory power of the regression was adequate, with an adjusted R^2 of .27, and the Durbin-Watson statistic showed no problem with autocorrelation.

The results from the three subsamples were very interesting. The speed of adjustment increased consistently over time, from .390 to .729, and the correlation with the market movement disappeared by the end of the sample. The coefficient differences were found to be significant at better than the 1% level using an F -test. These results suggest that as the market has developed, the importance of the equilibrium price in determining futures price changes has increased mark-

Table 4
Regression Results on the Dynamics of the Basis
 Regression Equation: $B_t - B_{t-1} = C_0 + C_1(B_t^e - B_{t-1}) + C_2(I_t - I_{t-1})$

Sample Period	Variable			\bar{R}^2	D.W.
	Constant	$(B_t^e - B_{t-1})$	$(I_t - I_{t-1})$		
Full Sample	.073 (1.63)	.430 (10.14)	.141 (4.81)	.27	2.34
First Third	-.095 (1.02)	.390 (5.59)	.176 (3.03)	.25	2.37
Middle Third	.134 (1.72)	.478 (6.24)	.203 (4.47)	.36	2.21
Last Third	.308 (4.74)	.729 (8.25)	.027 (0.65)	.38	2.01

NOTES: The hypothesis that the coefficients are equal across subsamples is rejected by an F test at the 1% critical level.

t-statistics are in parentheses.

edly, and overreaction of futures prices to changes in the spot index has diminished, perhaps to zero. Finally, we should note that if more than 70% of the deviation from the theoretical price is eliminated each day, it points to noise as the primary explanation for mispricing rather than any specific factor which was left out of the calculation for the equilibrium price.

V. Summary

This paper has presented a number of results relating to different aspects of basis risk and the behavior of the basis for Standard and Poor's 500 futures. In the final section it will be useful to summarize what we have found and discuss the implications for hedging strategy and contract design in this market.

A striking result from our comparison of hedging performance with different cash portfolios was that unsystematic risk is very important. Risk reduction for diversified portfolios of small stocks, like the Amex index portfolio, was rather limited. This suggests that valid hedging applications involving individual stocks or small portfolios may not be easy to find, especially when the intended hedge duration is short. A more effective hedge may be achievable with a more specialized instrument, such as an industry group index option or future, or an individual stock option. Another observation was that the risk minimizing hedge ratio was in all cases smaller than the beta of the portfolio being hedged, contrary to what has been suggested elsewhere.

In considering the sources of basis risk in a hedge of the S&P portfolio itself, we found that dividend risk was not an important factor, while hedge duration and time to expiration of the futures contract were, to some extent. One day hedges were subject to substantially more basis risk than one week hedges, but there did not seem to be any further improvement for extending the holding period to four weeks. Time to expiration became a factor when the futures contract had more than two months to maturity. The loss of hedging effectiveness

at that horizon suggests that there might be some value in having more frequent expiration dates than every three months, as is the case currently.

With regard to the pricing of stock index futures, we found that the significant underpricing that was widely remarked in the early months of trading seems to have disappeared, and that deviations from the theoretical pricing relation have diminished. This implies that the underpricing did not reflect an equilibrium differential, such as a factor like the value of the tax timing option would cause. Rather, it was a transitory phenomenon associated with the early stages of trading in the new market.

We found other evidence indicating that the character of the futures market has changed over time in the fact that the speed of adjustment of the actual futures price toward the theoretical level has increased. At present about 70 percent of a discrepancy is eliminated in one day. The earlier "overreaction" of the futures price to a change in the spot index also now seems to be gone. Overall, the evidence points to the conclusion that the stock index futures market is now fairly efficient and becoming moreso with time.

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